$B \to K^* \tau^+ \tau^-$ decay in the general two Higgs doublet model including the neutral Higgs boson effects

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Abstract

We study the CP violating asymmetry, the forward-backward asymmetry of the lepton pair and the CP asymmetry in the forward backward asymmetry for the exclusive decay $B \to K^* \tau^+ \tau^-$ in the general two Higgs doublet model including the neutral Higgs boson effects. We analyse the dependencies of these quantities on the model III parameters, We found that the physical parameters studied above are at the order of the magnitude 1% and neutral Higgs boson effects are detectable for large values of the coupling $\bar{\xi}^D_{N,\tau\tau}$.

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1 Introduction

Rare B decays are induced by flavor changing neutral currents (FCNC) at loop level in the Standard model (SM) and they are rich phenomenologically. They open a window for the determination of free parameters in the SM and also the investigation of the physics beyond, such as, two Higgs Doublet model (2HDM), Minimal Supersymmetric extension of the SM (MSSM) [1], etc. The experimental work for rare B decays continue at SLAC (BaBar), KEK (BELLE), B-Factories, DESY (HERA-B) and this stimulates the theoretical effort on them.

The exclusive $B \to K^* l^+ l^-$ process is an important candidate among rare B decays. Since its branching ratio (Br) predicted in the SM is large, there is a strong hope that this physical quantity is measured in the near future. The inclusive decay which induces $B \to K^* l^+ l^-$ process is $b \to s l^+ l^-$ transition and it is extensively studied in the literature, in the framework of the SM, 2HDM and MSSM. In [2]-[15], $b \to s l^+ l^-$ process is studied for light lepton pairs, namely $l = e, \mu$. In this case, the neutral Higgs boson (NHB) effects can be neglected since those contributions are proportional to the light lepton masses or corresponding Yukawa couplings. However for $l = \tau$, the NHB effects give sizable contributions. In [16, 17] $B \to X_s \tau^+ \tau^-$ process was studied in the model I, II versions the 2HDM and it was shown that NHB effects are important for large values of $tan\beta$. Currently the inclusive $b \to s l^+ l^-$ decay was studied in the model III version of 2HDM [18] and it was observed that NHB effects can give considerable contribution if the Yukawa interaction between τ lepton and neutral Higgs bosons is large.

The theoretical analysis of exclusive decays is difficult due to the hadronic form factors which contain uncertainities. However, their experimental investigation is easier compared to the one for the inclusive decays. The calculation of physical observables in the hadronic level needs non-perturbative methods. In the literature there are different studies based on different approches, such as relativistic quark model by lightfront formalism [15], chiral theory [19], three point QCD sum rules method [20], effective heavy quark theory [21] and light cone QCD sum rules [22, 23].

With the measured upper limit 5.2×10^{-6} (4.0×10^{-6}) for the Br of the decay $B^+ \to K^+ \mu^+ \mu^-$ ($B^0 \to K^{*0} \mu^+ \mu^-$) [24], the process $B \to K^* l^+ l^-$ have reached great interest. There are various studies on these decays in the SM, SM with fourth generation, multi Higgs doublet models, MSSM and in a model independent way, in the literature [2]-[15] and [19]-[36].

The CP violating effect is an important physical quantity to ensure the information about the free parameters of the model used. Since the CP violation for $B \to K^* l^+ l^-$ decay almost vanishes in the SM due to the unitarity of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and smallness of the term $V_{ub} V_{us}^*$, we have a chance to investigate the physics beyond the SM by searching the

CP violating effects. In the model III version of the 2HDM, there is a new source for CP violation, namely complex Yukawa couplings. In [26, 27], CP violating effects due to new phases in the model III and three Higgs doublet model, $3HDM(O_2)$, were studied and it was observed that a considerable CP asymmetry was obtained.

In this work, we study the exclusive $B \to K^* \tau^+ \tau^-$ decay in the general 2HDM , so-called model III, by including NHB effects. We use the quark level effective Hamiltonian which is calculated in [18] and investigate the A_{CP} and the forward-backward asymmetry (A_{FB}) of the lepton pair for the process underconsideration. Further, we calculate the CP asymmetry in A_{FB}, $(A_{CP}(A_{FB}))$ and observe that it can be measured in the forthcoming experiments.

The paper is organized as follows: In Section 2, we present the leading order (LO) QCD corrected effective Hamiltonian and the corresponding matrix element for the inclusive $b \to s\tau^+\tau^-$ decay, including NHB effects. Further, we give the matrix element for the exclusive $B \to K^* \tau^+ \tau^-$ decay and the explicit expressions for A_{FB}, A_{CP} and $A_{CP}(A_{FB})$. Section 3 is devoted to the analysis of the dependencies of A_{FB}, A_{CP} and $A_{CP}(A_{FB})$ on the CP parameter $sin\,\theta$, Yukawa coupling $\bar{\xi}^D_{N,\tau\tau}$ and the mass ratio $\frac{m_{h^0}}{m_{A^0}}$ and to the discussion of our results. In Appendices, we give the explicit forms of the operators appearing in the effective Hamiltonian, the corresponding Wilson coefficients and the form factors existing in the hadronic matrix elements.

2 The exclusive $B \to K^* \tau^+ \tau^-$ decay in the model III including NHB effects.

In the model III, the flavour changing neutral currents in the tree level are permitted and various new parameters, such as Yukawa couplings, masses of new Higgs bosons, exist. Yukawa couplings describe the interaction of fermions with gauge bosons. Our starting point is the inclusive $b \to s\tau^+\tau^-$ process which induces the exclusive $B \to K^* \tau^+ \tau^-$ decay and the corresponding Yukawa interaction is given by

$$\begin{aligned}\mathcal{L}_Y &= \eta^U_{ij} \bar{Q}_{iL} \tilde{\phi}_1 U_{jR} + \eta^D_{ij} \bar{Q}_{iL} \phi_1 D_{jR} + \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \phi_2 D_{jR} \\ &+ \eta^D_{kl} \bar{l}_{kL} \phi_1 E_{lR} + \xi^D_{kl} \bar{l}_{kL} \phi_2 E_{lR} + h.c. \ , \qquad (1)\end{aligned}$$

where i, j (k, l) are family indices of quarks (leptons), L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_m for $m = 1, 2$, are the two scalar doublets, Q_{iL} (l_{kL}) are quark (lepton) doublets, U_{jR}, D_{jR} (E_{lR}) are the corresponding quark (lepton) singlets, $\eta^{U,D}_{ij}$ and $\xi^{U,D}_{ij}$ are the matrices of the

Yukawa couplings which have complex entries in general. Here ϕ_1 and ϕ_2 are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array}\right) + \left(\begin{array}{c} \sqrt{2}\chi^+ \\ i\chi^0 \end{array}\right)\right] ; \phi_2 = \frac{1}{\sqrt{2}}\left(\begin{array}{c} \sqrt{2}H^+ \\ H_1 + iH_2 \end{array}\right) . \tag{2}$$

with the vacuum expectation values,

$$< \phi_1 > = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v \end{array}\right) ; < \phi_2 > = 0 . \tag{3}$$

With this choice, the SM particles can be collected in the first doublet and the new particles in the second one. Further, we take H_1, H_2 as the mass eigenstates h^0, A^0 respectively. Note that, at tree level, there is no mixing among CP even neutral Higgs bosons, namely the SM one, H^0, and beyond, h^0.

The part which produce FCNC at tree level is

$$\mathcal{L}_{Y,FC} = \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^{D}_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + \xi^{D}_{kl} \bar{l}_{kL} \phi_2 E_{lR} + h.c. \; . \tag{4}$$

In eq.(4) the couplings $\xi^{U,D}$ for the FC charged interactions are

$$\begin{aligned} \xi^{U}_{ch} &= \xi_{neutral} \, V_{CKM} \, , \\ \xi^{D}_{ch} &= V_{CKM} \, \xi_{neutral} \, , \end{aligned} \tag{5}$$

where $\xi^{U,D}_{neutral}$ is defined by the expression

$$\xi^{U(D)}_{N} = (V^{U(D)}_{R(L)})^{-1} \xi^{U,(D)} V^{U(D)}_{L(R)} \; . \tag{6}$$

and $\xi^{U,D}_{neutral}$ is denoted as $\xi^{U,D}_{N}$. Here the charged couplings are the linear combinations of neutral couplings multiplied by V_{CKM} matrix elements (see [37] for details).

At this stage we would like to present the calculation of the matrix element for the inclusive $b \to s\tau^+\tau^-$ decay, briefly. The procedure is the following:

- The calculation of the full theory including the NHB effects which comes from the interactions of neutral Higgs bosons H^0, h^0 and A^0 with τ lepton.

- Overcoming the logarithmic divergences by using the on-shell renormalization scheme. Here the renormalized vertex function is taken as

$$\Gamma^{Ren}_{neutr}(p^2) = \Gamma^0_{neutr}(p^2) + \Gamma^C_{neutr}, \tag{7}$$

  with the renormalization condition

$$\Gamma^{Ren}_{neutr}(p^2 = m^2_{neutr}) = 0, \tag{8}$$

and the counter terms are obtained. Here the phrase $neutr$ denotes the neutral Higgs bosons, H^0, h^0 and A^0 and p is the momentum transfer. Note that the self energy diagrams do not contribute in this scheme.

- Integrating out the heavy degrees of freedom, namely t quark, $W^\pm, H^\pm, H^0, h^0$, and A^0 bosons in the present case and obtaining the effective theory.

- Performing the QCD corrections through matching the full theory with the effective low energy one at the high scale $\mu = m_W$ and evaluating the Wilson coefficients from m_W down to the lower scale $\mu \sim O(m_b)$.

- Obtaining the effective Hamiltonian relevant for the process $b \to s\tau^+\tau^-$ which is given by

$$\mathcal{H}_{eff} = -4\frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} \left\{ \sum_i C_i(\mu) O_i(\mu) + \sum_i C_{Q_i}(\mu) Q_i(\mu) \right\} \,, \tag{9}$$

  where O_i are current-current ($i = 1, 2$), penguin ($i = 3, ..., 6$), magnetic penguin ($i = 7, 8$) and semileptonic ($i = 9, 10$) operators. Here, $C_i(\mu)$ are Wilson coefficients normalized at the scale μ and given in Appendix B. The additional operators $Q_i(i = 1, .., 10)$ are due to the NHB exchange diagrams and $C_{Q_i}(\mu)$ are their Wilson coefficients (see Appendices A and B) .

Therefore the QCD corrected amplitude for the inclusive $b \to s\tau^+\tau^-$ decay in the model III reads as,

$$\begin{aligned}
\mathcal{M} &= \frac{\alpha_{em} G_F}{\sqrt{2}\,\pi} V_{tb} V^*_{ts} \Big\{ C_9^{eff} (\bar{s}\gamma_\mu P_L b)\, \bar{\tau}\gamma_\mu \tau + C_{10} (\bar{s}\gamma_\mu P_L b)\, \bar{\tau}\gamma_\mu\gamma_5 \tau \\
&- 2C_7^{eff} \frac{m_b}{q^2} (\bar{s} i\sigma_{\mu\nu} q_\nu P_R b) \bar{\tau}\gamma_\mu \tau + C_{Q_1} (\bar{s} P_R b) \bar{\tau}\tau + C_{Q_2} (\bar{s} P_R b) \bar{\tau}\gamma_5 \tau \Big\} \,.
\end{aligned} \tag{10}$$

The matrix element for $B \to K^* l^+ l^-$ decay can be obtained by inserting the inclusive level effective Hamiltonian in eq. (9) between inital, B, and final, K^*, hadronic states. The necessary matrix elements in this calculation are $\langle K^* \,|\bar{s}\gamma_\mu(1 \pm \gamma_5) b|\, B\rangle$, $\langle K^* \,|\bar{s} i\sigma_{\mu\nu} q^\nu (1+\gamma_5) b|\, B\rangle$ and $\langle K^* \,|\bar{s}(1 \pm \gamma_5) b|\, B\rangle$. They are calculated by using some non-pertubative methods like QCD sum rules, light-cone QCD sum rules, etc., and using the parametrization of the form factors as in [20], the matrix element of the $B \to K^*\tau^+\tau^-$ decay is obtained as [22]:

$$\begin{aligned}
\mathcal{M} &= -\frac{G\alpha_{em}}{2\sqrt{2}\pi} V_{tb} V^*_{ts} \Big\{ \bar{\tau}\gamma^\mu \tau \left[2A\epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho_{K^*} q^\sigma + iB_1 \epsilon^*_\mu - iB_2 (\epsilon^* q)(p_B + p_{K^*})_\mu - iB_3 (\epsilon^* q) q_\mu \right] \\
&+ \bar{\tau}\gamma^\mu\gamma_5 \tau \left[2C\epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho_{K^*} q^\sigma + iD_1 \epsilon^*_\mu - iD_2 (\epsilon^* q)(p_B + p_{K^*})_\mu - iD_3 (\epsilon^* q) q_\mu \right] \\
&+ i\,\bar{\tau}\,\tau\, F(\epsilon^* q) + i\,\bar{\tau}\gamma_5 \tau\, G(\epsilon^* q) \Big\} \,,
\end{aligned} \tag{11}$$

where $\epsilon^{*\mu}$ is the polarization vector of K^* meson, p_B and p_{K^*} are four momentum vectors of B and K^* mesons, $q = p_B - p_{K^*}$. A, C, F and G, B_i and D_i, $i = 1, 2, 3$ are functions of Wilson coefficients and form factors of the relevant process. Their explicit forms are given in Appendix C.

Now we are ready to calculate the forward-backward asymmetry of lepton pair, CP-violating asymmetry and CP violating asymmetry in forward-backward asymmetry for the given process.

The forward-backward asymmetry A_{FB} of the lepton pair is a measurable physical quantity which provides important clues to test the theoretical models used. Using the definition of differential A_{FB}

$$A_{FB} = \frac{\int_0^1 dz \frac{d\Gamma}{dz} - \int_{-1}^0 dz \frac{d\Gamma}{dz}}{\int_0^1 dz \frac{d\Gamma}{dz} + \int_{-1}^0 dz \frac{d\Gamma}{dz}} \tag{12}$$

with $z = \cos\theta$, where θ is the angle between the momentum of B-meson and that of τ^- in the center of mass frame of the dileptons $\tau^+\tau^-$, we get

$$A_{FB} = \frac{\int \, ds\, E(s)}{\int \, ds\, D(s)}. \tag{13}$$

Here,

$$\begin{aligned}
E(s) &= 6\, m_B\, \lambda\, v^2 \Bigg\{ \frac{1}{m_b\, r\,(r-1)} \Bigg(4\, m_B\, m_\tau\, Re(C_7^{eff*}\, C_{Q_1}) \Big(m_b(\sqrt{r}-1)\, A_2\,(q^2) \\
&+ m_b(\sqrt{r}+1)\, A_1\,(q^2) - 2\, m_B\, s\, T_3\,(q^2) \Big) \Big((r-1)\,(3\,r - s + 1)\, T_2\,(q^2) + (r^2 + (s-1)^2 \\
&- 2\, r\,(s+1))\, T_3\,(q^2) \Big) \Bigg) \\
&- \frac{1}{m_b^2\, r\,(1+\sqrt{r})} \Bigg(m_B^2\, m_\tau\, Re(C_9^{eff*}\, C_{Q_1}) \Big(m_b(\sqrt{r}-1)\, A_2\,(q^2) + m_b(\sqrt{r}+1)\, A_1\,(q^2) \\
&- 2\, m_B\, s\, T_3\,(q^2) \Big) \Big((1+\sqrt{r})^2\,(r+s-1)\, A_1\,(q^2) + \lambda\, A_2\,(q^2) \Big) \Bigg) \\
&+ 8\, C_{10} \Bigg(-2\, m_B\, m_b\,(\sqrt{r}-1)\, Re\,(C_7^{eff})\, T_2\,(q^2)\, V\,(q^2) \\
&+ A_1\,(q^2) \Big(2\, m_b\, m_B(\sqrt{r}+1)\, Re\,(C_7^{eff})\, T_1\,(q^2) + m_B^2\, s\, Re(C_9^{eff})\, V(q^2) \Big) \Bigg) \Bigg\}
\end{aligned} \tag{14}$$

$$\begin{aligned}
D(s) &= \sqrt{\lambda}\, v \Bigg\{ \frac{32}{m_B\, s^2} m_b^2\, |C_7|^2\,(2\, m_\tau^2 + m_{B_s}^2\, s) \Bigg(\frac{2\, s}{r\,(r-1)} (1 + 3\, r - s)(T_2(q^2)\, T_3(q^2)\, \lambda) \\
&+ 8\, T_1^2(q^2)\, \lambda + \frac{T_3^2(q^2)\, s\, \lambda^2}{r\,(r-1)^2} + \frac{1}{r}\, T_2^2(q^2) \Big(12\,(r-1)^2\, r - (4\, r - s)\, \lambda \Big) \Bigg) \\
&+ \frac{2}{(1+\sqrt{r})^2\, r\, s} m_B |C_9^{eff}|^2 (2\, m_\tau^2 + m_{B_s}^2\, s) \Bigg(2\, A_1(q^2)\, A_2(q^2)\,(1+\sqrt{r})^2\,(r+s-1)\, \lambda \\
&+ A_1^2(q^2)\,(1+\sqrt{r})^4\,(12\, r\, s + \lambda) + \lambda\,(8\, r\, s\, V^2(q^2) + A_2^2(q^2)\, \lambda) \Bigg)
\end{aligned}$$

$$
\begin{aligned}
&+ \quad 2\,C_{10}^2\,m_B\Bigg(\frac{1}{r\,s}\Bigg(2\,A_1(q^2)\,A_2(q^2)(2\,m_\tau^2(r-2\,s-1)+m_{B_s}^2\,s\,(r+s-1))\lambda\Big)\\
&- \quad \frac{1}{m_b\,r}\Big(24\,m_B\,m_\tau^2(A_2(q^2)\,(-1+\sqrt{r})+A_1(q^2)\,(1+\sqrt{r}))\,T_3(q^2)\,\lambda\Big)\\
&+ \quad \frac{24}{m_b^2\,r}\,m_{B_s}^2\,m_\tau^2\,\lambda\,s\,T_3(q^2)+\frac{8}{(1+\sqrt{r})^2}(m_{B_s}^2\,s-4\,m_\tau^2)\,V^2(q^2)\,\lambda)\\
&+ \quad \frac{\lambda}{(1+\sqrt{r})^2\,r\,s}A_2^2(q^2)\Big(m_{B_s}^2\,s\,\lambda+2\,m_\tau^2(6\,s\,(1+r)-3\,s^2+\lambda))\Big)\\
&+ \quad \frac{1}{r\,s}\Big((1+\sqrt{r})^2\,A_1^2(q^2)(m_{B_s}^2\,s(12\,r\,s+\lambda)+m_\tau^2(-48\,r\,s+2\,\lambda))\Big)\Bigg)\\
&+ \quad \frac{3\,m_B^3\,\lambda}{m_b^4\,r}\,|C_{Q_1}|^2\,\Big(m_{B_s}^2\,s-4\,m_\tau^2\Big)\Big(A_2(q^2)\,m_b\,(\sqrt{r}-1)+A_1(q^2)\,m_b\,(\sqrt{r}+1)-2\,T_3(q^2)\,m_B\,s\Big)^2\\
&+ \quad \frac{3\,m_B^5\,\lambda}{m_b^2\,r}\,|C_{Q_2}|^2\,s\Big(A_2(q^2)\,(\sqrt{r}-1)+A_1(q^2)\,(\sqrt{r}+1)-2\,T_3(q^2)\,m_{B_s}^2\,\sqrt{r}\,s\Big)^2\\
&+ \quad \frac{1}{(1+\sqrt{r})^2\,s}\,Re(C_7^{eff*}\,C_9^{eff})16\,m_b\,(2\,m_\tau^2+m_{B_s}^2\,s)\Bigg(8(1+\sqrt{r})\,T_1(q^2)\,V(q^2)\,\lambda\\
&- \quad \frac{1}{(-1+\sqrt{r})\,r}\Big(A_2(q^2)\,(\lambda(r-1)(1+3\,r-s)\,T_2(q^2)+\lambda\,T_3(q^2))+A_1(q^2)\,(1+\sqrt{r})^2\\
&(\quad (r-1)\,T_2(q^2)\,(12(r-1)\,r-\lambda)+(r+s-1)\,T_3(q^2)\,\lambda)\Big)\Bigg)\\
&- \quad \frac{12}{m_b^2\,r}\,C_{10}\,Re(C_{Q_2})\,m_{B_s}^3\,m_\tau\Big(m_b\,((-1+\sqrt{r})\,A_2(q^2)+(1+\sqrt{r})\,A_1(q^2))\\
&- \quad 2\,m_B\,T_3(q^2)\Big)\Big(A_2(q^2)(1-\sqrt{r})-A_1(q^2)\,(1+\sqrt{r})+2\,m_{B_s}^2\,\sqrt{r}\,s\,T_3(q^2)\Big)\,\lambda\Big\}
\end{aligned}
\qquad (15)
$$

where $\lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $r = \frac{m_{K^*}^2}{m_B^2}$ and $s = \frac{q^2}{m_B^2}$.

The NHB effects bring new contribution to A_{FB} and we will study those contributions in the Discusssion part.

The complex Yukawa couplings are the possible source of CP violation in the model III. In our calculations we neglect all the Yukawa couplings, except $\bar{\xi}^U_{N,tt}$, $\bar{\xi}^D_{N,bb}$ and $\bar{\xi}^D_{N,\tau\tau}$ and choose $\bar{\xi}^D_{N,bb}$ complex, $\bar{\xi}^D_{N,bb} = |\bar{\xi}^D_{N,bb}|\,e^{i\theta}$ (see Discussion part). Therefore the CP violation comes from the Wilson coefficients C_7^{eff}, C_{Q_1} and C_{Q_2}. Using the definition of A_{CP}

$$
A_{CP} = \frac{\Gamma(B \to K^* \tau^+ \tau^-) - \Gamma(\bar{B} \to \bar{K}^* \tau^+ \tau^-)}{\Gamma(B \to K^* \tau^+ \tau^-) + \Gamma(\bar{B} \to \bar{K}^* \tau^+ \tau^-)}\ . \qquad (16)
$$

we get

$$
A_{CP} = \frac{\int\, ds\,\Omega(s)}{\int\, ds\,\Lambda(s)}\ . \qquad (17)
$$

where

$$
\Omega(s) \;=\; \frac{m_b\,\alpha_e^2\,G_F^2\,\lambda_t^2}{384\,\pi^5\,s\,(1+\sqrt{r})^2}\,v\,\sqrt{\lambda}\,Im(C_7^{eff})\,Im(C_9^{eff})\,(2\,m_\tau^2+m_{B_s}^2\,s)\Big\{8\,(\sqrt{r}+1)\,\lambda\,T_1(q^2)\,V(q^2)
$$

$$
\begin{aligned}
- & \frac{1}{r\,(\sqrt{r}-1)}\Bigg(\lambda\, A_2(q^2)\left((r-1)(3\,r-s+1)\,T_2(q^2)+\lambda\,T_3(q^2)\right) \\
+ & (\sqrt{r}+1)^2\,A_1(q^2)\left((r-1)\,T_2(q^2)(12\,(r-1)\,r-\lambda)+(r+s-1)\,\lambda\,T_3(q^2)\right)\Bigg)\Bigg\}
\end{aligned}
\tag{18}
$$

and

$$
\Lambda(s) = D(s) + D_{CP}(s)\ . \tag{19}
$$

Here $D_{CP}(s)$ is the CP conjugate of $D(s)$ which is defined as

$$
D_{CP}(s) = D(s)(\bar{\xi}^{D}_{N,bb} \to \bar{\xi}^{D*}_{N,bb})\ . \tag{20}
$$

The CP violating asymmetry in A_{FB} is also a measurable physical quantity and it can give strong clues for the physics beyond the SM. This quantity is defined as

$$
A_{CP}(A_{FB}) = \frac{A_{FB}-\bar{A}_{FB}}{A_{FB}+\bar{A}_{FB}}\ . \tag{21}
$$

where $\bar{A}_{FB}$ is the CP conjugate of A_{FB} and it is given as

$$
\bar{A}_{FB} = A_{FB}(\bar{\xi}^{D}_{N,bb} \to \bar{\xi}^{D*}_{N,bb})\ . \tag{22}
$$

Note that during the calculations of A_{CP}, A_{FB} and $A_{CP}(A_{FB})$ we take into account only the second resonance for the LD effects coming from the reaction $b \to s\psi_i \to s\tau^+\tau^-$, where $i = 1,..,6$ and divide the integration region for s into two parts : $\frac{4m_\tau^2}{m_B^2} \leq s \leq \frac{(m_{\psi_2}-0.02)^2}{m_B^2}$ and $\frac{(m_{\psi_2}+0.02)^2}{m_B^2} \leq s \leq 1$, where $m_{\psi_2} = 3.686\,GeV$ is the mass of the second resonance (see Appendix B for LD contributions).

3 Discussion

In the general 2HDM model, the number of free parameters, namely the masses of charged and neutral Higgs bosons and complex Yukawa couplings ($\xi^{U,D}_{ij}$), increases compared to the ones in the SM and model I (II) version of 2HDM. The arbitrariness of the numerical values of these parameters can be removed by using the restrictions coming from the experimental measurements.

Since the neutral Higgs bosons, h^0 and A^0, can give a large contribution to the coefficient C_7^{eff} (see the Appendix of [38] for details) which is in contradiction with the CLEO data [39],

$$
Br(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32)\,10^{-4}\ , \tag{23}
$$

we take $\bar{\xi}^{D}_{N,ib} \sim 0$ and $\bar{\xi}^{D}_{N,ij} \sim 0$, where the indices i, j denote d and s quarks . Further we use the constraints [37], coming from the $\Delta F = 2$ mixing (here $F = K, B_d, D$) decays, the ρ parameter [40],

and the measurement by CLEO Collaboration eq. (23), we get the condition for $\bar{\xi}_{Ntc}$, $\bar{\xi}_{Ntc} << \bar{\xi}^U_{Ntt}$ and take into account only the Yukawa couplings of quarks $\bar{\xi}^U_{N,tt}$ and $\bar{\xi}^D_{N,bb}$. We keep the Yukawa coupling $\bar{\xi}^D_{N,\tau\tau}$ free and increase this parameter to enhance the effects of neutral Higgs bosons.

In this section, we study the CP parameter $sin\theta$, the Yukawa coupling $\bar{\xi}^D_{N,\tau\tau}$ and the mass ratio $\frac{m_{h^0}}{m_{A^0}}$ dependencies of the A_{FB}, A_{CP} and $A_{CP}(A_{FB})$ of the exclusive decay $B \rightarrow K^* \tau^+ \tau^-$, restricting $|C_7^{eff}|$ in the region $0.257 \leq |C_7^{eff}| \leq 0.439$ due to the CLEO measurement, eq.(23) (see [37] for details). Our numerical calculations based on this restriction and throughout these calculations, we use the redefinition

$$\xi^{U,D} = \sqrt{\frac{4G_F}{\sqrt{2}}}\bar{\xi}^{U,D} \ ,$$

we take $|\frac{\bar{\xi}^U_{N,tt}}{\bar{\xi}^D_{N,bb}}| < 1$, the scale $\mu = m_b$, include the LD effects and use the input values given in Table (1).

Parameter	Value
m_τ	1.78 (GeV)
m_c	1.4 (GeV)
m_b	4.8 (GeV)
$\bar{\xi}^D_{N,bb}$	$40\, m_b$
α_{em}^{-1}	129
λ_t	0.04
m_t	175 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
m_{H^0}	150 (GeV)
m_{h^0}	70 (GeV)
$m_{H^\pm}$	400 (GeV)
Λ_{QCD}	0.225 (GeV)
$\alpha_s(m_Z)$	0.117
$sin\theta_W$	0.2325

Table 1: The values of the input parameters used in the numerical calculations.

In Fig. 1 we present $sin\theta$ dependence of A_{FB} without NHB effects, for $m_{A^0} = 80\, GeV$. Here A_{FB} lies in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). The solid straight line shows the SM contribution. In the model III without NHB effects, $|A_{FB}|$ is smaller compared to the one in the SM (0.195), for $C_7^{eff} > 0$, however it is possible to enhance it at the order of the magnitude 2% with increasing $sin\theta$. For $C_7^{eff} < 0$, A_{FB} is not sensitive to $sin\theta$ and the restriction region is narrow. For this case $|A_{FB}|$ can have slightly greater values compared to the SM one. Addition of NHB effects (see Fig. 2) reduces $|A_{FB}|$ for $C_7^{eff} > 0$ almost 30% compared to the one

without NHB effects. For $C_7^{eff} < 0$, the restriction region becomes narrow and A_{FB} reaches the SM prediction for small $sin\theta$.

Fig. 3 represent $\bar{\xi}^D_{N,\tau\tau}$ dependence of A_{FB} for $sin\theta = 0.5$ and $m_{A^0} = 80\,GeV$. $|A_{FB}|$ vanishes with increasing $\bar{\xi}^D_{N,\tau\tau}$ for $C_7^{eff} > 0$. For $C_7^{eff} < 0$, $|A_{FB}|$ does not vanish in the given region of $\bar{\xi}^D_{N,\tau\tau}$ and it stands less than the SM result.

Fig. 4 is devoted to the ratio $\frac{m_{h^0}}{m_{A^0}}$ dependence of A_{FB} for $sin\theta = 0.5$ and $\bar{\xi}^D_{N,\tau\tau} = 10\,m_\tau$. Increasing values of the ratio causes to increase $|A_{FB}|$ for both $C_7^{eff} > 0$ and $C_7^{eff} < 0$. If the masses of h^0 and A^0 are far from the degeneracy, $|A_{FB}|$ becomes small especially for $C_7^{eff} > 0$.

Figs 5-7 represent A_{CP} of the process $B \to K^* \tau^+ \tau^-$. In Fig. 5 we present $sin\theta$ dependence of A_{CP} without NHB effects, for $m_{A^0} = 80\,GeV$. Here A_{CP} lies in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). For $C_7^{eff} > 0$, A_{CP} is at the order of the magnitude of 1% for the intermediate values of $sin\theta$ and its sign does not change in the restriction region. However A_{CP} can have both signs, even vanish for $C_7^{eff} < 0$. With the addition of NHB effects (see Fig. 6) A_{CP} for $C_7^{eff} > 0$ decreases to almost one half of the value we get without NHB effects. For $C_7^{eff} < 0$ there is still a decrease in A_{CP}.This behavior can be seen from the expression eq. (18) since the numerator of the A_{CP} ratio is free from NHB effects and their additional contributions enter into the expression in the denominator part. Further the restriction regions becomes narrow.

Fig. 7 represent $\bar{\xi}^D_{N,\tau\tau}$ dependence of A_{CP} for $sin\theta = 0.5$ and $m_{A^0} = 80\,GeV$. A_{CP} is sensitive to the parameter $\bar{\xi}^D_{N,\tau\tau}$ and it decreases with increasing $\bar{\xi}^D_{N,\tau\tau}$ for $C_7^{eff} > 0$. However, for $C_7^{eff} < 0$, the dependence of A_{CP} to $\bar{\xi}^D_{N,\tau\tau}$ is weak.

The ratio $\frac{m_{h^0}}{m_{A^0}}$ dependence of A_{CP} for $sin\theta = 0.5$ and $\bar{\xi}^D_{N,\tau\tau} = 10\,m_\tau$ is presented in Fig. 8. As seen from the figure the sensitivity A_{CP} to the ratio is small, especially for $C_7^{eff} < 0$.

Finally, we present the CP violating asymmetry in A_{FB} in a series of figures (Figs. 9-11). Fig. 9 represent $sin\theta$ dependence of $A_{CP}(A_{FB})$ with NHB effects, for $m_{A^0} = 80\,GeV$ and $\bar{\xi}^D_{N,\tau\tau} = 10\,m_\tau$. $A_{CP}(A_{FB})$ is at the order of the magnitude of 1% for the intermediate values of $sin\theta$ for $C_7^{eff} > 0$. Its sign does not change in the restriction region similar to the A_{CP} of the process under consideration. However $A_{CP}(A_{FB})$ can have both signs, even vanish for $C_7^{eff} < 0$. $A_{CP}(A_{FB})$ is sensitive to the parameter $\bar{\xi}^D_{N,\tau\tau}$ especially for the large values of $\bar{\xi}^D_{N,\tau\tau}$ and $C_7^{eff} > 0$ (see Fig. 10). It can reach 10% for $\bar{\xi}^D_{N,\tau\tau} = 50\,GeV$. In the case $C_7^{eff} < 0$, $A_{CP}(A_{FB})$ is not sensitive to $\bar{\xi}^D_{N,\tau\tau}$ and it almost vanishes.

Fig. 11 is devoted to the ratio $\frac{m_{h^0}}{m_{A^0}}$ dependence of $A_{CP}(A_{FB})$ for $sin\theta = 0.5$ and $\bar{\xi}^D_{N,\tau\tau} = 10\,m_\tau$. Increasing values of the ratio causes to increase $|A_{CP}(A_{FB})|$ for $C_7^{eff} > 0$. With the increasing mass ratio of h^0 and A^0, $|A_{CP}(A_{FB})|$ can take large values. For $C_7^{eff} < 0$, $A_{CP}(A_{FB})$ is not sensitive to the mass ratio.

Now, we would like to summarize our results.

- $|A_{FB}|$ for the process under consideration is at the order of 10^{-2} and smaller compared to the SM one, for $C_7^{eff} > 0$. It can exceed the SM value (0.195) for $C_7^{eff} < 0$. Addition of NHB effects decreases its magnitude by 30% (slightly) for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). A_{FB} is sensitive to the parameters $sin\theta$, $\bar{\xi}^D_{N,\tau\tau}$ and $\frac{m_{h^0}}{m_{A^0}}$ especially for $C_7^{eff} > 0$. Its magnitude decreases (increases) with increasing values of $\bar{\xi}^D_{N,\tau\tau}$ ($\frac{m_{h^0}}{m_{A^0}}$).

- $|A_{CP}|$ is at the order of 10^{-2}. Addition of NHB effects decreases its magnitude by 50% (slightly) for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). It has the same sign in the restriction region $C_7^{eff} > 0$ and it can take both signs for $C_7^{eff} < 0$. A_{CP} is sensitive to the parameters $sin\theta$, $\bar{\xi}^D_{N,\tau\tau}$ especially for $C_7^{eff} > 0$. It decreases with increasing values of $\bar{\xi}^D_{N,\tau\tau}$. The sensitivity of A_{CP} to the ratio $\frac{m_{h^0}}{m_{A^0}}$ is weak.

- $A_{CP}(A_{FB})$ is at the order of the magnitude of 1% for the intermediate values of $sin\theta$ for $C_7^{eff} > 0$. It has the same sign in the restriction region $C_7^{eff} > 0$ and it can take both signs for $C_7^{eff} < 0$. $A_{CP}(A_{FB})$ is sensitive to the parameters $\bar{\xi}^D_{N,\tau\tau}$ and $\frac{m_{h^0}}{m_{A^0}}$ for $C_7^{eff} > 0$. It increases with increasing values of $\bar{\xi}^D_{N,\tau\tau}$, even reach to 10%. Further the increasing values of the ratio $\frac{m_{h^0}}{m_{A^0}}$ causes to increase $|A_{CP}(A_{FB})|$.

Therefore, the experimental investigation of A_{FB} and A_{CP} and $A_{CP}(A_{FB})$ ensure a crucial test for new physics effects beyond the SM and also the sign of C_7^{eff}.

A The operator basis

The operator basis in the 2HDM (model III) for our process is [16, 41, 42]

$$\begin{aligned}
O_1 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}), \\
O_2 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}), \\
O_3 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\beta}), \\
O_4 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}), \\
O_5 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\beta}), \\
O_6 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}), \\
O_7 &= \frac{e}{16\pi^2}\bar{s}_\alpha\sigma_{\mu\nu}(m_b R + m_s L)b_\alpha\mathcal{F}^{\mu\nu}, \\
O_8 &= \frac{g}{16\pi^2}\bar{s}_\alpha T^a_{\alpha\beta}\sigma_{\mu\nu}(m_b R + m_s L)b_\beta\mathcal{G}^{a\mu\nu}\ , \\
O_9 &= \frac{e}{16\pi^2}(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{\tau}\gamma^\mu\tau)\ , \\
O_{10} &= \frac{e}{16\pi^2}(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{\tau}\gamma^\mu\gamma_5\tau)\ , \\
Q_1 &= \frac{e^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R)\,(\bar{\tau}\tau) \\
Q_2 &= \frac{e^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R)\,(\bar{\tau}\gamma_5\tau) \\
Q_3 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L\, q^\beta_R) \\
Q_4 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R\, q^\beta_L) \\
Q_5 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L\, q^\alpha_R) \\
Q_6 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R\, q^\alpha_L) \\
Q_7 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\,\sigma^{\mu\nu}\, b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L\,\sigma_{\mu\nu} q^\beta_R) \\
Q_8 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\,\sigma^{\mu\nu}\, b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R\,\sigma_{\mu\nu} q^\beta_L) \\
Q_9 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\,\sigma^{\mu\nu}\, b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L\,\sigma_{\mu\nu} q^\alpha_R) \\
Q_{10} &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\,\sigma^{\mu\nu}\, b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R\,\sigma_{\mu\nu} q^\alpha_L)
\end{aligned} \tag{24}$$

where α and β are $SU(3)$ colour indices and $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively. Note that there are also flipped chirality partners of these operators, which can be obtained by interchanging L and R in the basis given above in model III. However, we do not present them here since corresponding Wilson coefficients are negligible.

B The Initial values of the Wilson coefficients.

The initial values of the Wilson coefficients for the relevant process in the SM are [41]

$$
\begin{aligned}
C^{SM}_{1,3,\ldots 6}(m_W) &= 0 \; , \\
C^{SM}_{2}(m_W) &= 1 \; , \\
C^{SM}_{7}(m_W) &= \frac{3x_t^3-2x_t^2}{4(x_t-1)^4}\ln x_t + \frac{-8x_t^3-5x_t^2+7x_t}{24(x_t-1)^3} \; , \\
C^{SM}_{8}(m_W) &= -\frac{3x_t^2}{4(x_t-1)^4}\ln x_t + \frac{-x_t^3+5x_t^2+2x_t}{8(x_t-1)^3} \; , \\
C^{SM}_{9}(m_W) &= -\frac{1}{sin^2\theta_W}B(x_t) + \frac{1-4\sin^2\theta_W}{\sin^2\theta_W}C(x_t) - D(x_t) + \frac{4}{9}, \; , \\
C^{SM}_{10}(m_W) &= \frac{1}{\sin^2\theta_W}(B(x_t)-C(x_t)) \; , \\
C^{SM}_{Q_i}(m_W) &= 0 \quad i=1,..,10 \; .
\end{aligned} \tag{25}
$$

and for the additional part due to charged Higgs bosons are

$$
\begin{aligned}
C^{H}_{1,\ldots 6}(m_W) &= 0 \, , \\
C^{H}_{7}(m_W) &= Y^2\, F_1(y_t) \, + \, XY\, F_2(y_t) \; , \\
C^{H}_{8}(m_W) &= Y^2\, G_1(y_t) \, + \, XY\, G_2(y_t) \; , \\
C^{H}_{9}(m_W) &= Y^2\, H_1(y_t) \; , \\
C^{H}_{10}(m_W) &= Y^2\, L_1(y_t) \; ,
\end{aligned} \tag{26}
$$

where

$$
\begin{aligned}
X &= \frac{1}{m_b} \quad \left(\bar{\xi}^{D}_{N,bb} + \bar{\xi}^{D}_{N,sb}\frac{V_{ts}}{V_{tb}}\right) \quad , \\
Y &= \frac{1}{m_t} \quad \left(\bar{\xi}^{U}_{N,tt} + \bar{\xi}^{U}_{N,tc}\frac{V^*_{cs}}{V^*_{ts}}\right) \quad ,
\end{aligned} \tag{27}
$$

The NHB effects bring new operators and the corresponding Wilson coefficients read as

$$
C^{A^0}_{Q_2}((\bar{\xi}^{U}_{N,tt})^3) = \frac{\bar{\xi}^{D}_{N,\tau\tau}(\bar{\xi}^{U}_{N,tt})^3 m_b y_t(\Theta_5(y_t)z_A - \Theta_1(z_A,y_t))}{32\pi^2 m^2_{A^0} m_t \Theta_1(z_A,y_t)\Theta_5(y_t)},
$$

$$C^{A^0}_{Q_2}((\bar{\xi}^U_{N,tt})^2) = \frac{\bar{\xi}^D_{N,\tau\tau}(\bar{\xi}^U_{N,tt})^2\bar{\xi}^D_{N,bb}}{32\pi^2 m^2_{A^0}}\Big(\frac{(y_t(\Theta_1(z_A,y_t)-\Theta_5(y_t)(xy+z_A))-2\Theta_1(z_A,y_t)\Theta_5(y_t)\ln[\frac{z_A\Theta_5(y_t)}{\Theta_1(z_A,y_t)}]}{\Theta_1(z_A,y_t)\Theta_5(y_t)}\Big),$$

$$\begin{aligned} C^{A^0}_{Q_2}(\bar{\xi}^U_{N,tt}) &= \frac{g^2\bar{\xi}^D_{N,\tau\tau}\bar{\xi}^U_{N,tt}m_b x_t}{64\pi^2 m^2_{A^0}m_t}\Bigg(\frac{2}{\Theta_5(x_t)}-\frac{xyx_t+2z_A}{\Theta_1(z_A,x_t)}-2\ln[\frac{z_A\Theta_5(x_t)}{\Theta_1(z_A,x_t)}] \\ &-xyx_ty_t(\frac{(x-1)x_t(y_t/z_A-1)-(1+x)y_t)}{(\Theta_6-(x-y)(x_t-y_t))(\Theta_3(z_A)+(x-y)(x_t-y_t)z_A)}-\frac{x(y_t+x_t(1-y_t/z_A))-2y_t}{\Theta_6\Theta_3(z_A)})\Bigg)\end{aligned}$$

$$C^{A^0}_{Q_2}(\bar{\xi}^D_{N,bb}) = \frac{g^2\bar{\xi}^D_{N,\tau\tau}\bar{\xi}^D_{N,bb}}{64\pi^2 m^2_{A^0}}\Big(1-\frac{x_t^2y_t+2y(x-1)x_ty_t-z_A(x_t^2+\Theta_6)}{\Theta_3(z_A)}+\frac{x_t^2(1-y_t/z_A)}{\Theta_6}+2\ln[\frac{z_A\Theta_6}{\Theta_2(z_A,x)}]\Big)$$

$$\begin{aligned} C^{H^0}_{Q_1}((\bar{\xi}^U_{N,tt})^2) &= \frac{g^2(\bar{\xi}^U_{N,tt})^2 m_b m_\tau}{64\pi^2 m^2_{H^0}m_t^2}\Bigg(\frac{x_t(1-2y)y_t}{\Theta_5(y_t)}+\frac{(-1+2\cos^2\theta_W)(-1+x+y)y_t}{\cos^2\theta_W\Theta_4(y_t)} \\ &+\frac{z_H(\Theta_1(z_H,y_t)xy_t+\cos^2\theta_W\,(-2x^2(-1+x_t)yy_t^2+xx_tyy_t^2-\Theta_8z_H))}{\cos^2\theta_W\Theta_1(z_H,y_t)\Theta_7}\Bigg),\end{aligned} \tag{28}$$

$$\begin{aligned} C^{H^0}_{Q_1}(\bar{\xi}^U_{N,tt}) &= \frac{g^2\bar{\xi}^U_{N,tt}\bar{\xi}^D_{N,bb}m_\tau}{64\pi^2 m^2_{H^0}m_t}\Bigg(\frac{(-1+2\cos^2\theta_W)\,y_t}{\cos^2\theta_W\,\Theta_4(y_t)}-\frac{x_ty_t}{\Theta_5(y_t)}+\frac{x_ty_t(xy-z_H)}{\Theta_1(z_H,y_t)} \\ &+\frac{(-1+2\cos^2\theta_W)y_tz_H}{\cos^2\theta_W\Theta_7}-2x_t\,\ln\left[\frac{\Theta_5(y_t)z_H}{\Theta_1(z_H,y_t)}\right]\Bigg),\end{aligned}$$

$$\begin{aligned} C^{H^0}_{Q_1}(g^4) &= -\frac{g^4m_bm_\tau x_t}{128\pi^2m^2_{H^0}m_t^2}\Bigg(-1+\frac{(-1+2x)x_t}{\Theta_5(x_t)+y(1-x_t)}+\frac{2x_t(-1+(2+x_t)y)}{\Theta_5(x_t)} \\ &-\frac{4\cos^2\theta_W(-1+x+y)+x_t(x+y)}{\cos^2\theta_W\Theta_4(x_t)}+\frac{x_t(x(x_t(y-2z_H)-4z_H)+2z_H)}{\Theta_1(z_H,x_t)} \\ &+\frac{y_t((-1+x)x_tz_H+\cos^2\theta_W((3x-y)z_H+x_t(2y(x-1)-z_H(2-3x-y))))}{\cos^2\theta_W(\Theta_3(z_H)+x(x_t-y_t)z_H)} \\ &+2\,(x_t\ln\left[\frac{\Theta_5(x_t)z_H}{\Theta_1(z_H,x_t)}\right]+\ln\left[\frac{x(y_t-x_t)z_H-\Theta_3(z_H)}{(\Theta_5(x_t)+y(1-x_t)y_tz_H}\right])\Bigg),\end{aligned}$$

$$C^{h_0}_{Q_1}((\bar{\xi}^U_{N,tt})^3) = -\frac{\bar{\xi}^D_{N,\tau\tau}(\bar{\xi}^U_{N,tt})^3m_by_t}{32\pi^2m^2_{h^0}m_t\Theta_1(z_h,y_t)\Theta_5(y_t)}\Big(\Theta_1(z_h,y_t)(2y-1)+\Theta_5(y_t)(2x-1)z_h\Big)$$

$$C^{h_0}_{Q_1}((\bar{\xi}^U_{N,tt})^2) = \frac{\bar{\xi}^D_{N,\tau\tau}(\bar{\xi}^U_{N,tt})^2}{32\pi^2m^2_{h^0}}\Bigg(\frac{(\Theta_5(y_t)z_h(y_t-1)(x+y-1)-\Theta_1(z_h,y_t)(\Theta_5(y_t)+y_t)}{\Theta_1(z_h)\Theta_5(y_t)}-2\ln\left[\frac{z_h\Theta_5(y_t)}{\Theta_1(z_h)}\right]\Bigg)$$

$$\begin{aligned} C^{h^0}_{Q_1}(\bar{\xi}^U_{N,tt}) &= -\frac{g^2\bar{\xi}^D_{N,\tau\tau}\bar{\xi}^U_{N,tt}m_bx_t}{64\pi^2m^2_{h^0}m_t}\Bigg(\frac{2(-1+(2+x_t)y)}{\Theta_5(x_t)}-\frac{x_t(x-1)(y_t-z_h)}{\Theta_2'(z_h)}+2\ln\left[\frac{z_h\Theta_5(x_t)}{\Theta_1(z_h,x_t)}\right] \\ &+\frac{x(x_t(y-2z_h)-4z_h)+2z_h}{\Theta_1(z_h,x_t)}-\frac{(1+x)y_tz_h}{xyx_ty_t+z_h((x-y)(x_t-y_t)-\Theta_6)} \\ &+\frac{\Theta_9+y_tz_h((x-y)(x_t-y_t)-\Theta_6)(2x-1)}{z_h\Theta_6(\Theta_6-(x-y)(x_t-y_t))}+\frac{x(y_tz_h+x_t(z_h-y_t))-2y_tz_h}{\Theta_2(z_h)}\Bigg),\end{aligned}$$

$$C^{h^0}_{Q_1}(\bar{\xi}^D_{N,bb}) = -\frac{g^2\bar{\xi}^D_{N,\tau\tau}\bar{\xi}^D_{N,bb}}{64\pi^2 m^2_{h^0}}\left(\frac{yx_ty_t(xx_t^2(y_t-z_h)+\Theta_6 z_h(x-2))}{z_h\Theta_2(z_h)\Theta_6}+2\ln\left[\frac{\Theta_6}{x_ty_t}\right]+2\ln\left[\frac{x_ty_tz_h}{\Theta_2(z_h)}\right]\right)$$

where

$$\begin{aligned}
\Theta_1(\omega,\lambda) &= -(-1+y-y\lambda)\omega - x(y\lambda+\omega-\omega\lambda)\\
\Theta_2(\omega) &= (x_t+y(1-x_t))y_t\omega - xx_t(yy_t+(y_t-1)\omega)\\
\Theta_2'(\omega) &= \Theta_2(\omega, x_t\leftrightarrow y_t)\\
\Theta_3(\omega) &= (x_t(-1+y)-y)y_t\omega + xx_t(yy_t+\omega(-1+y_t))\\
\Theta_4(\omega) &= 1-x+x\omega\\
\Theta_5(\lambda) &= x+\lambda(1-x)\\
\Theta_6 &= (x_t+y(1-x_t))y_t + xx_t(1-y_t)\\
\Theta_7 &= (y(y_t-1)-y_t)z_H + x(yy_t+(y_t-1)z_H)\\
\Theta_8 &= y_t(2x^2(1+x_t)(y_t-1)+x_t(y(1-y_t)+y_t)+x(2(1-y+y_t)\\
&+ x_t(1-2y(1-y_t)-3y_t)))\\
\Theta_9 &= -x_t^2(-1+x+y)(-y_t+x(2y_t-1))(y_t-z_h) - x_ty_tz_h(x(1+2x)-2y)\\
&+ y_t^2(x_t(x^2-y(1-x))+(1+x)(x-y)z_h)
\end{aligned} \tag{29}$$

and

$$x_t=\frac{m_t^2}{m_W^2}\quad,\quad y_t=\frac{m_t^2}{m_{H^\pm}}\quad,\quad z_H=\frac{m_t^2}{m_{H^0}^2}\quad,\quad z_h=\frac{m_t^2}{m_{h^0}^2}\quad,\quad z_A=\frac{m_t^2}{m_{A^0}^2}\quad,$$

The explicit forms of the functions $F_{1(2)}(y_t)$, $G_{1(2)}(y_t)$, $H_1(y_t)$ and $L_1(y_t)$ in eq.(26) are given as

$$\begin{aligned}
F_1(y_t) &= \frac{y_t(7-5y_t-8y_t^2)}{72(y_t-1)^3}+\frac{y_t^2(3y_t-2)}{12(y_t-1)^4}\ln y_t\ ,\\
F_2(y_t) &= \frac{y_t(5y_t-3)}{12(y_t-1)^2}+\frac{y_t(-3y_t+2)}{6(y_t-1)^3}\ln y_t\ ,\\
G_1(y_t) &= \frac{y_t(-y_t^2+5y_t+2)}{24(y_t-1)^3}+\frac{-y_t^2}{4(y_t-1)^4}\ln y_t\ ,\\
G_2(y_t) &= \frac{y_t(y_t-3)}{4(y_t-1)^2}+\frac{y_t}{2(y_t-1)^3}\ln y_t\ ,\\
H_1(y_t) &= \frac{1-4sin^2\theta_W}{sin^2\theta_W}\ \frac{xy_t}{8}\left[\frac{1}{y_t-1}-\frac{1}{(y_t-1)^2}\ln y_t\right]\\
&- y_t\left[\frac{47y_t^2-79y_t+38}{108(y_t-1)^3}-\frac{3y_t^3-6y_t+4}{18(y_t-1)^4}\ln y_t\right]\ ,\\
L_1(y_t) &= \frac{1}{sin^2\theta_W}\ \frac{xy_t}{8}\left[-\frac{1}{y_t-1}+\frac{1}{(y_t-1)^2}\ln y_t\right]\ .
\end{aligned} \tag{30}$$

Finally, the initial values of the coefficients in the model III are

$$\begin{aligned}
C_i^{2HDM}(m_W) &= C_i^{SM}(m_W) + C_i^{H}(m_W), \\
C_{Q_1}^{2HDM}(m_W) &= \int_0^1 dx \int_0^{1-x} dy\, (C_{Q_1}^{H^0}((\bar{\xi}^U_{N,tt})^2) + C_{Q_1}^{H^0}(\bar{\xi}^U_{N,tt}) + C_{Q_1}^{H^0}(g^4) + C_{Q_1}^{h^0}((\bar{\xi}^U_{N,tt})^3) \\
&+ C_{Q_1}^{h^0}((\bar{\xi}^U_{N,tt})^2) + C_{Q_1}^{h^0}(\bar{\xi}^U_{N,tt}) + C_{Q_1}^{h^0}(\bar{\xi}^D_{N,bb})), \\
C_{Q_2}^{2HDM}(m_W) &= \int_0^1 dx \int_0^{1-x} dy\, (C_{Q_2}^{A^0}((\bar{\xi}^U_{N,tt})^3) + C_{Q_2}^{A^0}((\bar{\xi}^U_{N,tt})^2) + C_{Q_2}^{A^0}(\bar{\xi}^U_{N,tt}) + C_{Q_2}^{A^0}(\bar{\xi}^D_{N,bb})) \\
C_{Q_3}^{2HDM}(m_W) &= \frac{m_b}{m_\tau \sin^2\theta_W}(C_{Q_1}^{2HDM}(m_W) + C_{Q_2}^{2HDM}(m_W)) \\
C_{Q_4}^{2HDM}(m_W) &= \frac{m_b}{m_\tau \sin^2\theta_W}(C_{Q_1}^{2HDM}(m_W) - C_{Q_2}^{2HDM}(m_W)) \\
C_{Q_i}^{2HDM}(m_W) &= 0\ ,\ i = 5, ..., 10.
\end{aligned} \tag{31}$$

Here, we present C_{Q_1} and C_{Q_2} in terms of the Feynmann parameters x and y since the integrated results are extremely large. Using these initial values, we can calculate the coefficients $C_i^{2HDM}(\mu)$ and $C_{Q_i}^{2HDM}(\mu)$ at any lower scale in the effective theory with five quarks, namely u, c, d, s, b similar to the SM case [13, 34, 38, 42].

The Wilson coefficients playing the essential role in this process are $C_7^{2HDM}(\mu)$, $C_9^{2HDM}(\mu)$, $C_{10}^{2HDM}(\mu)$, $C_{Q_1}^{2HDM}(\mu)$ and $C_{Q_2}^{2HDM}(\mu)$. For completeness, in the following we give their explicit expressions.

$$C_7^{eff}(\mu) = C_7^{2HDM}(\mu) + Q_d\,(C_5^{2HDM}(\mu) + N_c\, C_6^{2HDM}(\mu))\ ,$$

where the LO QCD corrected Wilson coefficient $C_7^{LO,2HDM}(\mu)$ is given by

$$\begin{aligned}
C_7^{LO,2HDM}(\mu) &= \eta^{16/23} C_7^{2HDM}(m_W) + (8/3)(\eta^{14/23} - \eta^{16/23}) C_8^{2HDM}(m_W) \\
&+ C_2^{2HDM}(m_W) \sum_{i=1}^{8} h_i \eta^{a_i}\ ,
\end{aligned} \tag{32}$$

and $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, h_i and a_i are the numbers which appear during the evaluation [13].

$C_9^{eff}(\mu)$ contains a perturbative part and a part coming from LD effects due to conversion of the real $\bar{c}c$ into lepton pair $\tau^+\tau^-$:

$$C_9^{eff}(\mu) = C_9^{pert}(\mu) + Y_{reson}(s)\ , \tag{33}$$

where

$$
\begin{aligned}
C_9^{pert}(\mu) &= C_9^{2HDM}(\mu) \\
&+ h(z,s)\left(3C_1(\mu)+C_2(\mu)+3C_3(\mu)+C_4(\mu)+3C_5(\mu)+C_6(\mu)\right) \\
&- \frac{1}{2}h(1,s)\left(4C_3(\mu)+4C_4(\mu)+3C_5(\mu)+C_6(\mu)\right) \\
&- \frac{1}{2}h(0,s)\left(C_3(\mu)+3C_4(\mu)\right)+\frac{2}{9}\left(3C_3(\mu)+C_4(\mu)+3C_5(\mu)+C_6(\mu)\right)\ ,
\end{aligned} \tag{34}
$$

and

$$
\begin{aligned}
Y_{reson}(s) &= -\frac{3}{\alpha_{em}^2}\kappa\sum_{V_i=\psi_i}\frac{\pi\Gamma(V_i\to\tau^+\tau^-)m_{V_i}}{q^2-m_{V_i}+im_{V_i}\Gamma_{V_i}} \\
&\quad \left(3C_1(\mu)+C_2(\mu)+3C_3(\mu)+C_4(\mu)+3C_5(\mu)+C_6(\mu)\right).
\end{aligned} \tag{35}
$$

In eq.(33), the functions $h(u,s)$ are given by

$$
\begin{aligned}
h(u,s) &= -\frac{8}{9}\ln\frac{m_b}{\mu}-\frac{8}{9}\ln u+\frac{8}{27}+\frac{4}{9}x \\
&\quad -\frac{2}{9}(2+x)|1-x|^{1/2}\begin{cases}\left(\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right|-i\pi\right), & \text{for } x\equiv\frac{4u^2}{s}<1\\ 2\arctan\frac{1}{\sqrt{x-1}}, & \text{for } x\equiv\frac{4u^2}{s}>1,\end{cases}
\end{aligned} \tag{36}
$$

$$
h(0,s) = \frac{8}{27}-\frac{8}{9}\ln\frac{m_b}{\mu}-\frac{4}{9}\ln s+\frac{4}{9}i\pi\ , \tag{37}
$$

with $u=\frac{m_c}{m_b}$. The phenomenological parameter κ in eq. (35) is taken as 2.3. In eqs. (30) and (35), the contributions of the coefficients $C_1(\mu)$, ..., $C_6(\mu)$ are due to the operator mixing.

Finally, the Wilson coefficients $C_{Q_1}(\mu)$ and $C_{Q_2}(\mu)$ are given by [16]

$$
C_{Q_i}(\mu)=\eta^{-12/23}\,C_{Q_i}(m_W)\ ,\ i=1,2\ . \tag{38}
$$

C The form factors for the decay $B\to K^* l^+ l^-$

The structure functions appearing in eq. (11) are

$$
\begin{aligned}
A &= -C_9^{eff}\frac{V}{m_B+m_{K^*}}-4C_7^{eff}\frac{m_b}{q^2}T_1\ , \\
B_1 &= -C_9^{eff}(m_B+m_{K^*})A_1-4C_7^{eff}\frac{m_b}{q^2}(m_B^2-m_{K^*}^2)T_2\ , \\
B_2 &= -C_9^{eff}\frac{A_2}{m_B+m_{K^*}}-4C_7^{eff}\frac{m_b}{q^2}\left(T_2+\frac{q^2}{m_B^2-m_{K^*}^2}T_3\right)\ , \\
B_3 &= -C_9^{eff}\frac{2m_{K^*}}{q^2}(A_3-A_0)+4C_7^{eff}\frac{m_b}{q^2}T_3\ , \\
C &= -C_{10}\frac{V}{m_B+m_{K^*}}\ ,
\end{aligned}
$$

$$
\begin{aligned}
D_1 &= -C_{10}(m_B + m_{K^*})A_1 \ , \\
D_2 &= -C_{10}\frac{A_2}{m_B + m_{K^*}} \ , \\
D_3 &= -C_{10}\frac{2m_{K^*}}{q^2}(A_3 - A_0) \ , \\
F &= C_{Q_1}\frac{2m_{K^*}}{m_b}A_0 \ , \\
G &= C_{Q_2}\frac{2m_{K^*}}{m_b}A_0 \ ,
\end{aligned} \tag{39}
$$

We use the q^2 dependent expression which is calculated in the framework of light-cone QCD sum rules in [23] to calculate the hadronic form factors $V, \ A_1, \ A_2, \ A_0, \ T_1, \ T_2$ and T_3:

$$
F(q^2) = \frac{F(0)}{1 - a_F\frac{q^2}{m_B^2} + b_F(\frac{q^2}{m_B^2})^2} \ , \tag{40}
$$

where the values of parameters $F(0)$, a_F and b_F are listed in 2.

	$F(0)$	a_F	b_F
A_1	0.34 ± 0.05	0.60	-0.023
A_2	0.28 ± 0.04	1.18	0.281
V	0.46 ± 0.07	1.55	0.575
T_1	0.19 ± 0.03	1.59	0.615
T_2	0.19 ± 0.03	0.49	-0.241
T_3	0.13 ± 0.02	1.20	0.098

Table 2: The values of parameters existing in eq.(40) for the various form factors of the transition $B \to K^*$.

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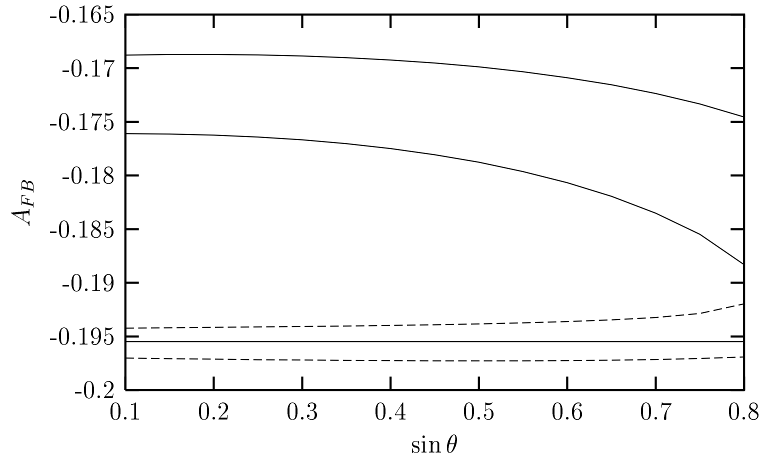

Figure 1: A_{FB} as a function of $sin\theta$ for $m_{A^0} = 80\, GeV$ without NHB effects. Here A_{FB} is restricted in the region between solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Straight line corresponds to the SM contribution.

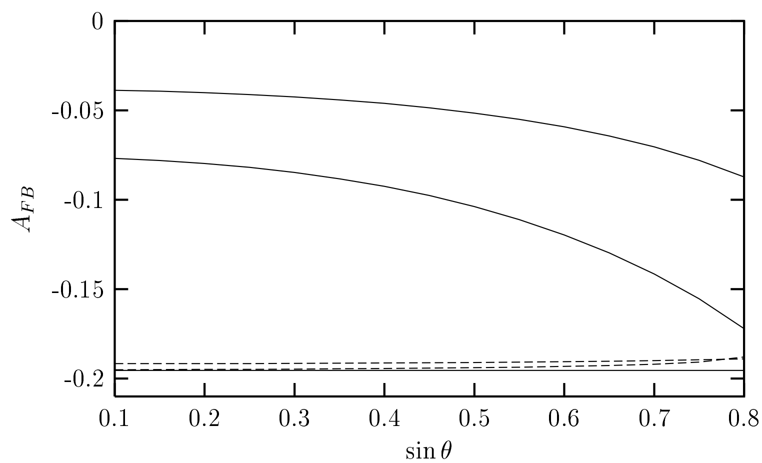

Figure 2: The same as Fig.1, but for $\bar{\xi}^{D}_{N,\tau\tau} = 10\, m_\tau$ and including NHB effects.

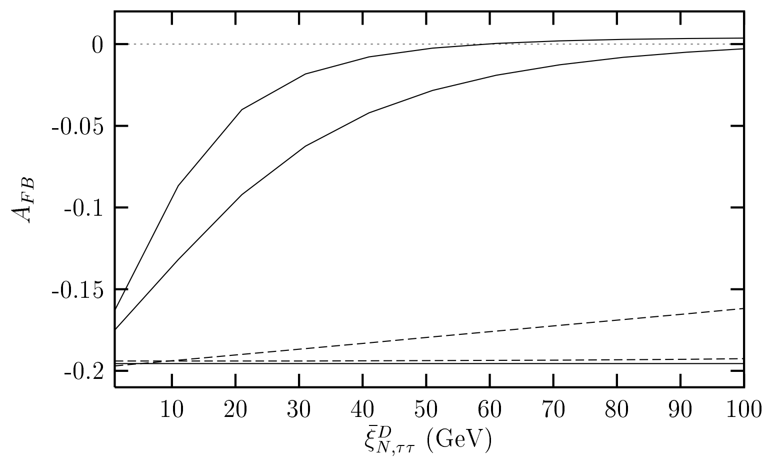

Figure 3: A_{FB} as a function of $\bar{\xi}^{D}_{N,\tau\tau}$ for $sin\theta = 0.5$ and $m_{A^0} = 80\, GeV$. Here A_{FB} is restricted in the region between solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Straight line corresponds to the SM contribution.

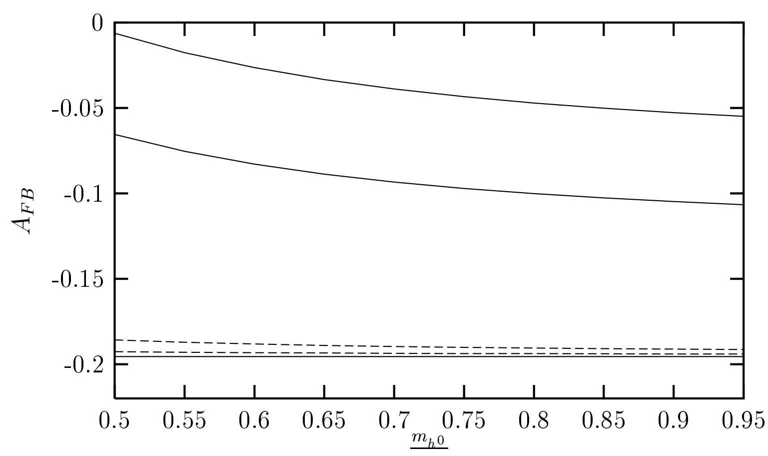

Figure 4: A_{FB} as a function of $\frac{m_{h^0}}{m_{A^0}}$ for $sin\theta = 0.5$ and $\bar{\xi}^{D}_{N,\tau\tau} = 10\, m_\tau$. Here A_{FB} is restricted in the region between solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Straight line corresponds to the SM contribution.

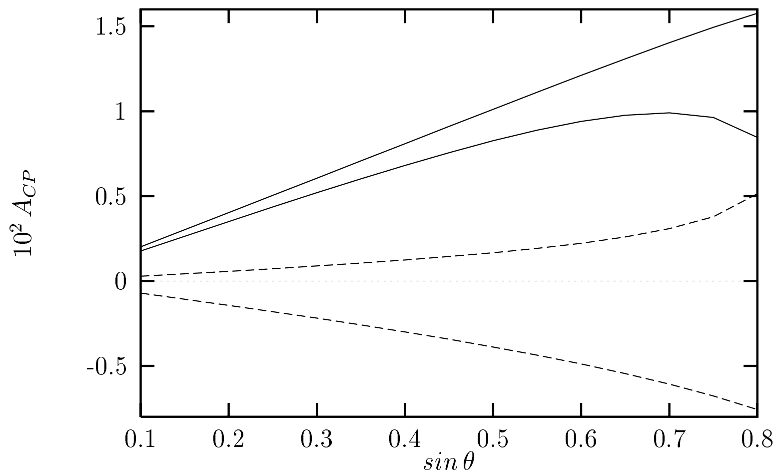

Figure 5: The same as Fig. 1 but for A_{CP}.

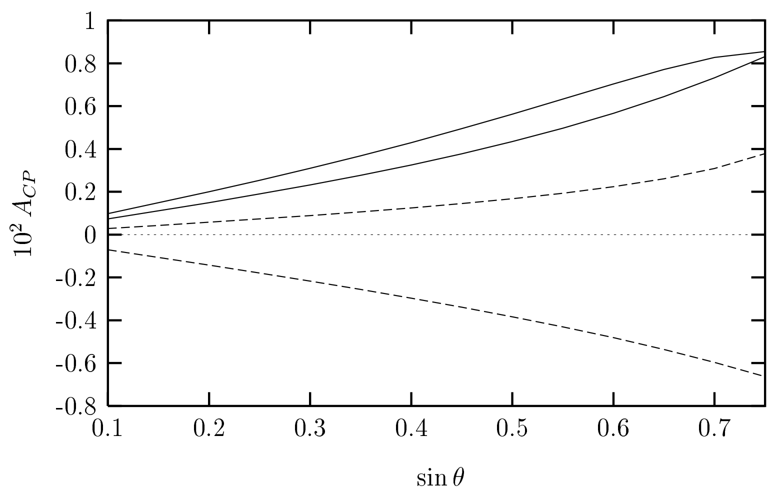

Figure 6: The same as Fig. 2 but for A_{CP}.

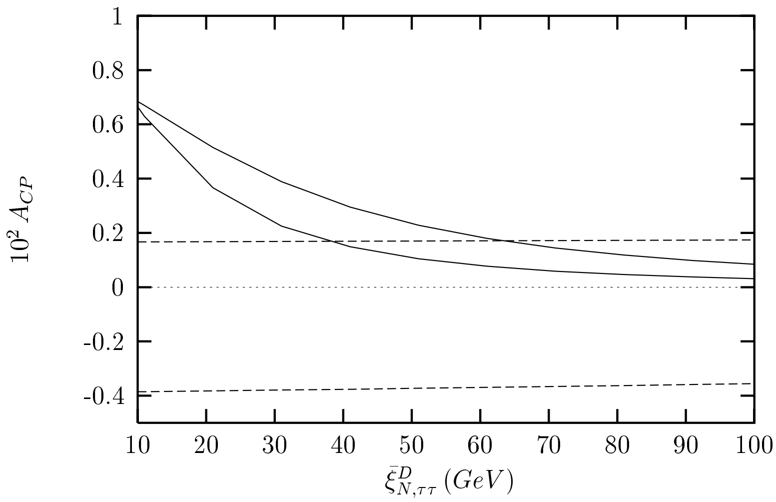

Figure 7: The same as Fig. 3 but for A_{CP}.

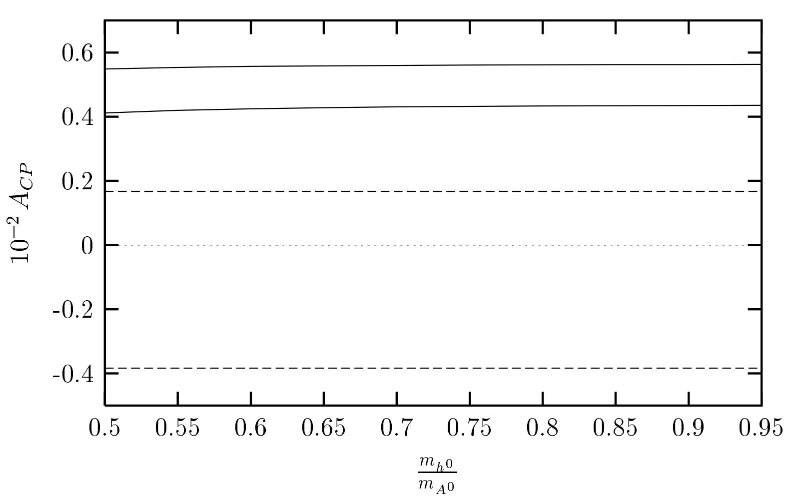

Figure 8: The same as Fig. 4 but for A_{CP}.

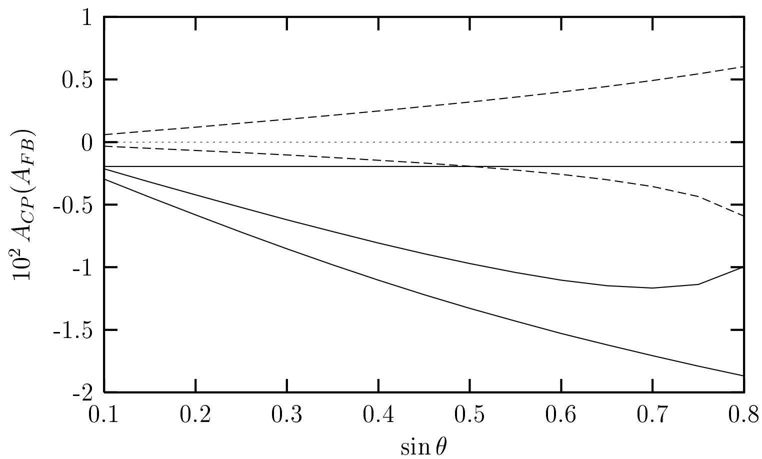

Figure 9: The same as Fig. 2 but for $A_{CP}(A_{FB})$.

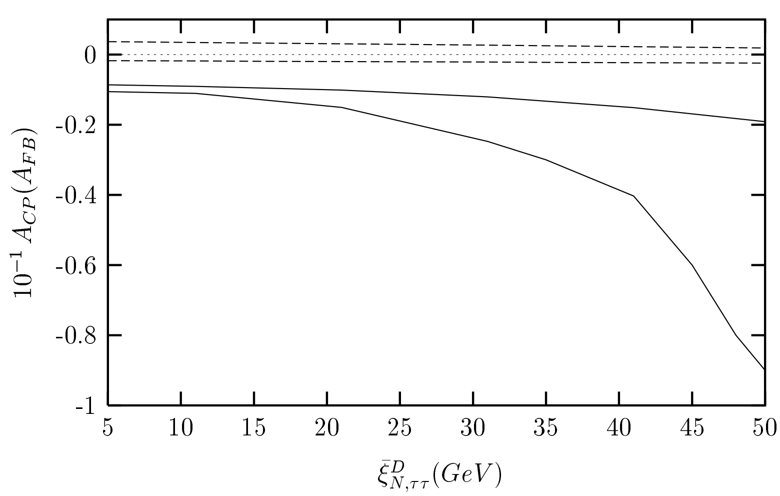

Figure 10: The same as Fig. 3 but for $A_{CP}(A_{FB})$.

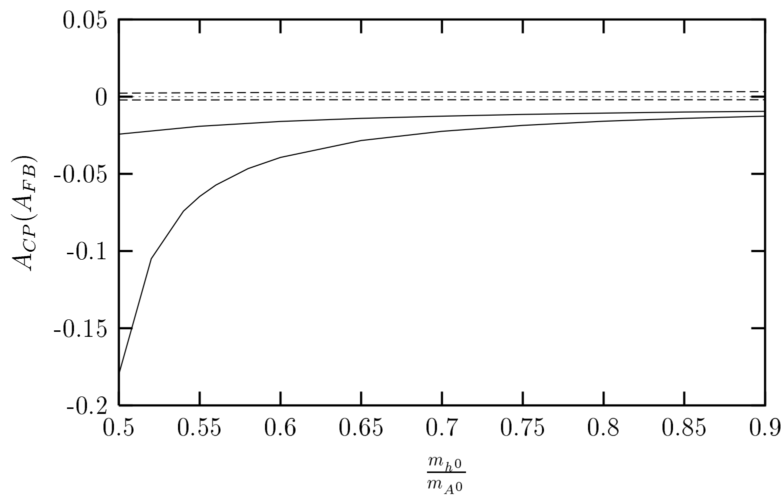

Figure 11: The same as Fig. 4 but for $A_{CP}(A_{FB})$.